

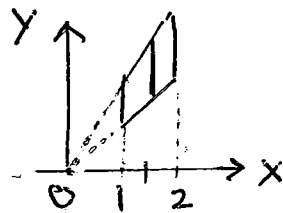
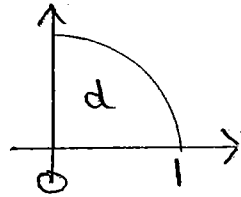
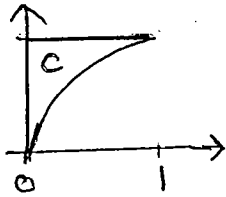
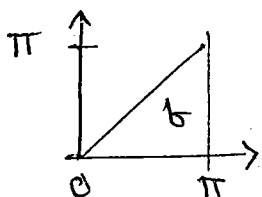
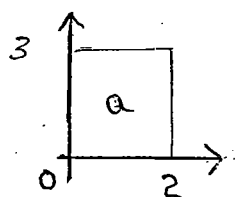
CALCULUS II TUTORIAL 10: SOLUTIONS

$$\textcircled{1} (a) \int_0^3 \int_0^2 (4-y^2) dy dx = \int_0^3 \left[4y - \frac{1}{3}y^3 \right]_0^2 dx = \frac{16}{3} [x]_0^3 = 16.$$

$$(b) \int_0^\pi \int_0^x x \sin y dy dx = \int_0^\pi [-x \cos y]_0^x dx = \int_0^\pi (x - x \cos x) dx \\ = \left[\frac{1}{2}x^2 - x \sin x \right]_0^\pi + \int_0^\pi \sin x dx = \frac{1}{2}\pi^2 - [\cos x]_0^\pi = \frac{1}{2}\pi^2 + 2$$

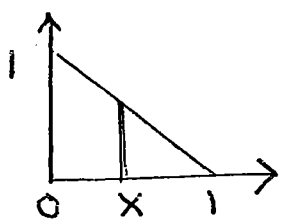
$$(c) \int_0^1 \int_0^{y^2} 3y^3 e^{xy} dx dy = \int_0^1 [3y^3 e^{xy}]_0^{y^2} dy \\ = \int_0^1 (3y^3 e^{y^3} - 3y^3) dy = [e^{y^3} - y^3]_0^1 = e - 2$$

$$(d) \int_0^1 \int_0^{\sqrt{1-x^2}} 8y dy dx = \int_0^1 [4y^2]_0^{\sqrt{1-x^2}} dx = \int_0^1 (4 - 4x^2) dx \\ = [4x - \frac{4}{3}x^3]_0^1 = \frac{8}{3}$$



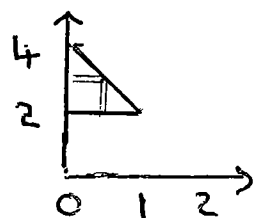
$$\textcircled{2} (a) \int_1^2 \int_x^{2x} \frac{x}{y} dy dx = \int_1^2 x [\ln y]_x^{2x} dx \\ = \int_1^2 x \ln 2 dx = \frac{1}{2} \ln 2 [x^2]_1^2 = \frac{3}{2} \ln 2$$

$$(b) \int_0^1 \int_0^{1-x} (x^2 + y^2) dy dx = \int_0^1 \left[x^2 y + \frac{1}{3} y^3 \right]_0^{1-x} dx \\ = \int_0^1 \left\{ x^2 - x^3 + \frac{1}{3}(1-x)^3 \right\} dx = \left[\frac{1}{2}x^3 - \frac{1}{4}x^4 - \frac{1}{2}(1-x)^4 \right]_0^1 = \frac{1}{6}$$



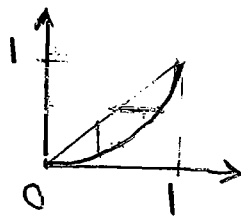
$$\textcircled{3} (a) \int_0^1 \int_2^{4-2x} dy dx = \int_2^4 \int_0^{\frac{4-y}{2}} dx dy$$

not required $\left[= \int_2^4 \left[x \right]_0^{\frac{4-y}{2}} dy = \int_2^4 \left(2 - \frac{1}{2}y \right) dy = \left[2y - \frac{1}{4}y^2 \right]_2^4 = 1 \right]$



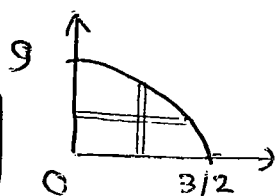
$$(b) \int_0^1 \int_y^{\sqrt{y}} dx dy = \int_0^1 \int_{x^2}^x dy dx$$

$$\rightarrow \left[= \int_0^1 [y]_{x^2}^x dx = \int_0^1 (x - x^2) dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^1 = \frac{1}{6} \right]$$



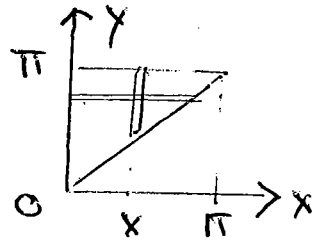
$$(c) \int_0^{3/2} \int_0^{9-4x^2} dy dx = \int_0^9 \int_0^{\frac{\sqrt{9-y}}{2}} dx dy$$

$$\rightarrow \left[= \int_0^9 \left[x \right]_0^{\frac{\sqrt{9-y}}{2}} dy = \int_0^9 \frac{\sqrt{9-y}}{2} dy = \frac{1}{3} \left[(9-y)^{3/2} \right]_0^9 = 9 \right]$$



$$\textcircled{4} \text{ (a) } \int_0^\pi \int_x^\pi \frac{\sin y}{y} dy dx = \int_0^\pi \int_0^y \frac{\sin y}{y} dx dy$$

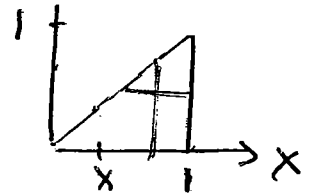
$$= \int_0^\pi [x]_0^y \frac{\sin y}{y} dy = \int_0^\pi \sin y dy = [\cos y]_\pi^0 = 2$$



$$\text{ (b) } \int_0^1 \int_y^1 x^2 e^{xy} dx dy = \int_0^1 \int_0^x x^2 e^{xy} dy dx$$

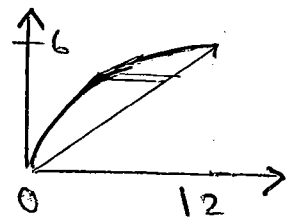
$$= \int_0^1 [x e^{xy}]_0^x dx = \int_0^1 (x e^{x^2} - x) dx$$

$$= \left[\frac{1}{2} e^{x^2} - \frac{1}{2} x^2 \right]_0^1 = \frac{1}{2} e - 1$$



$$\textcircled{5} \int_0^6 \int_{y^2/3}^{2y} dx dy = \int_0^6 [x]_{y^2/3}^{2y} dy = \int_0^6 (2y - \frac{y^2}{3}) dy$$

$$= [y^2 - \frac{1}{9} y^3]_0^6 = 12. \text{ Note: } 2y = \frac{y^2}{3} \Rightarrow y=0,6$$

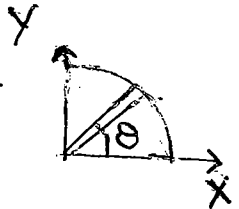


$$\textcircled{6} \int_0^2 \int_0^2 (x^2 + y^2) dx dy = \int_0^2 [\frac{1}{3} x^3 + y^2 x]_0^2 dy$$

$$= \int_0^2 (\frac{8}{3} + 2y^2) dy = [\frac{8}{3} y + \frac{2}{3} y^3]_0^2 = \frac{32}{3} \Rightarrow \langle x^2 + y^2 \rangle = \frac{8}{3}$$

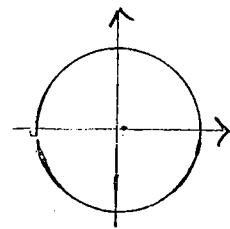
$$\textcircled{7} \text{ (a) } \int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy = \int_0^{\pi/2} \int_0^1 r^2 r dr d\theta$$

$$= \int_0^{\pi/2} d\theta \int_0^1 r^3 dr = \frac{\pi}{2} [\frac{1}{4} r^4]_0^1 = \frac{\pi}{8}$$



$$\text{ (b) } \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2 + y^2 + 1) dx dy \rightarrow$$

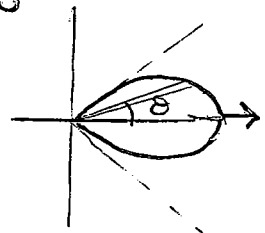
$$= \int_{-\pi}^{\pi} d\theta \int_0^1 \ln(r^2 + 1) r dr = 2\pi \frac{1}{2} [(r^2 + 1) \ln(r^2 + 1) - r^2]_0^1 = (\ln 4 - 1)\pi$$



$$\textcircled{8} \text{ Area} = \int_{-\pi/6}^{\pi/6} \int_0^{12 \cos 3\theta} r dr d\theta = \int_{-\pi/6}^{\pi/6} [\frac{1}{2} r^2]_0^{12 \cos 3\theta} d\theta$$

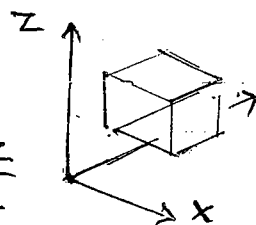
$$= \int_{-\pi/6}^{\pi/6} 72 \cos^2 3\theta d\theta = \int_{-\pi/6}^{\pi/6} (36 + 36 \cos 6\theta) d\theta$$

$$= [36\theta + 6 \sin 6\theta]_{-\pi/6}^{\pi/6} = 12\pi$$



⑨ (a) $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dx dy dz = \int_1^e \frac{dx}{x} \int_1^e \frac{dy}{y} \int_1^e \frac{dz}{z}$

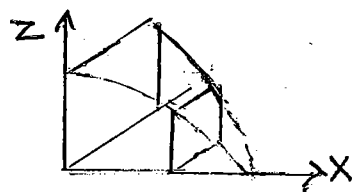
$= [\ln x]_1^e [\ln y]_1^e [\ln z]_1^e = 1$



(b) $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2}} dz dy dx$

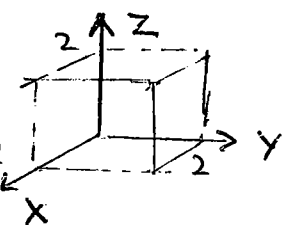
$= \int_0^3 \int_0^{\sqrt{9-x^2}} [z]_0^{\sqrt{9-x^2}} dy dx$

$= \int_0^3 (9-x^2) dx = [9x - \frac{1}{3}x^3]_0^3 = 18$



⑩ $\int_0^2 \int_0^2 \int_0^2 (x^2 + 9) dx dy dz = \int_0^2 \int_0^2 [\frac{1}{3}x^3 + 9x]_0^2 dy dz$

$= \int_0^2 \int_0^2 \frac{62}{3} dy dz = \frac{248}{3} \Rightarrow \langle x^2 + 9 \rangle = \frac{248}{3 \times 8} = \frac{31}{3}$



⑪ $X = u \cos v, Y = u \sin v \Rightarrow \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \cos v & -u \sin v \\ \sin v & u \cos v \end{vmatrix} = u$

⑫ $u = 3x + 2y, v = x + 4y \Rightarrow x = \frac{2u-v}{5}, y = \frac{3v-u}{10}$

$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{2}{5} & -\frac{1}{5} \\ -\frac{1}{10} & \frac{3}{10} \end{vmatrix} = \frac{1}{10}$

$y = -\frac{3}{2}x + 1 \Rightarrow u = 2, y = -\frac{3}{2}x + 3 \Rightarrow u = 6$

$y = -\frac{1}{4}x \Rightarrow v = 0, y = -\frac{1}{4}x + 1 \Rightarrow v = 4$

$\iint_R (3x^2 + 14xy + 8y^2) dx dy = \iint_R (3x+2y)(x+4y) dx dy$

$= \int_0^4 \int_2^6 uv du dv = \int_0^4 u du \int_2^6 v dv = [\frac{1}{2}u^2]_0^4 [\frac{1}{2}v^2]_2^6 = 128$

